

# **THE DYNAMIC INTERACTIONS OF GROWTH AND DEPRECIATION**

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## ABSTRACT

The determinants of depreciation function are neglected concept in the theory of economic growth. This study investigates interactions between capital accumulation and the evolution of depreciation functions in a dynamical Solow model. The model introduces non-linear depreciation functions into the theory of growth. Two effects of “congestion” and “recycling” influence depreciation towards opposing outcomes: (1) poverty (depreciation) trap: the entire investments can only cover the increasing depreciation; (2) a type of endogenous /sustainable growth that capital accumulation and technological progress give rise to an everlasting growth via decreasing depreciation power. Therefore, poverty trap can be avoided and prosperous sustainable/ endogenous growth might be achieved if congestion of obsolete physical capital is prevented and depreciation power is lowered. To this end, diversification of economic activities, waste management, recycling, and investment in higher-quality durable goods are recommended.

Keywords: Capital, depreciation, economic growth, obsolescence, dynamical models.

JEL Classification: E23, O10, O44, O57

## Introduction

Depreciation or obsolescence of capital is a neglected factor in the literature of economic growth. It is evident that the quality (durability) of infrastructures and other types of physical capital are remarkably varied among countries. Gylfason and Zoega (2007) acknowledge the importance of this factor:

*“Around the world, differences in the quality of housing, capital and infrastructure are at least as evident as are differences in the quantity of such capital. ... We see vast differences in the quality of housing and other infrastructure. Whether of their own deserts or not, some nations are clearly endowed with physical capital of higher quality than others even if their national income accounts often do not reflect these important differences.”*

In this paper quality and depreciation are dual concepts and higher quality dictates higher durability or longer productive life of physical capital. The expected lifespan of any durable

product (capital) is a decision variable. Our objective of this paper is to shed light on the long-term severe consequences of early stage investment in inferior capital. The policy advice is to lower the depreciation power by avoiding congestion and establishment of “reverse supply chain management” that completes the recycling cycle of capital and durable goods after their useful life.

## The Model

We introduce a dynamical model in the framework of Slow-Swan with a non-linear depreciation function. Inspired by Ramsey, Cass and Koopsman model, we establish a dynamical system to explain the interactions between non-linear depreciation function and capital accumulation mechanism.

In the literature of modern growth theory, depreciation function is always assumed a linear of per capita capital, which is simply calculated by multiplying a constant exogenous parameter,  $\Delta$ , by the per capita capital stock:  $\Delta k$ . However, it is evident that higher stages of growth give rise to more diversified economies which in turn results in more efficient chains of recycling activities of physical capital. Therefore, as capital accumulates, more recycling activities, which used to be out of reach due to prohibitive levels of economies of scale and/or scope, are becoming feasible. Also, more diversification establishes networks of businesses that facilitate recycling of different types of physical capital. Furthermore, more developed economies afford to financially support and legally protect innovations, which diversify economic activities. We refer to this impact of diversification based on economies of scale, network externalities, and maintenance-related innovations as “recycling effect.” The more diversified an economy, the more effective capital preservation, thus the lower the depreciation rate.

Thus capital accumulation will impose a mitigating recycling effect on the depreciation rate,  $\Delta$ . We model this plausible negative effect by considering a  $\Delta$  as a function of  $k$ . Higher development reduces the power of per capita capital stock in this function to a level lower than one  $\beta_1 \leq 0$ . Therefore instead of having a constant  $\Delta$ , now we have:

$$\Delta = k^{\beta_1}, \quad \beta_1 \leq 0$$

On the other hand, at all stages of growth, the physical capital invested in the past could be prohibitive and exponentially increases the depreciation rates. For instance, an investment in a two-lane road facilitates growth at the early stages; however, later on, congestion might simultaneously reduce productivity and increase depreciation of vehicles. In fact, obsolescence of early- invested capital can be costly and impede economic growth. Replacement of obsolete capital follows its own economic rules: the marginal benefits of replacement should exceed the marginal costs. Thus economic agents act based on their private interests. In this mindset, of course, innovations are recommended due to their positive impact on productivity. However, the overlooked fact is that the public costs of replacements or their negative externalists to the whole

economy. At very early stages of development, these costs are negligible; nevertheless, they are becoming cumbersome as obsolete capitals are congested in the intermediate stages of growth. We refer to these effects as “congestion effects,” which exponentially increases  $\Delta$ :

$$\Delta = \delta k^{\beta_2}, \quad \beta_2 \geq 0$$

It is noteworthy that any restriction imposed by prior investments could be surpassed if financial resources are available. Therefore, at higher levels of development, the amplifying impact of congestion effect on depreciation would be limited.

Combining the two effects of “recycling” and “congestion” results in a non-linear depreciation function that the power of  $\beta$  might assume any value below, equal, or above one:

$$\Delta = \delta k^{\beta}, \quad \beta = \beta_1 + \beta_2$$

In the above equation,  $\beta$  represents combining effects of congestion and recycling. At the early stages of growth, accumulation of capital gradually drives congestion effects; however, this effect reaches its pick at later stages of intermediate development. The recycling effects lag the congestion effects and appear when networks of diversified economic activities facilitate effective recycling in the economy. To put it the other way, recycling effects require development of “recycling institutions” to complete “recycling supply chain management.”

In our model, there exists a level of capital,  $k^*$ , below which congestion effects dominate and above which recycling effects dominate. As a result, time derivative of the power of capital in the depreciation function  $\dot{\beta}$  is a function of per capita capital such as:

$$\dot{\beta} = g(k)$$

where:

$$g(k) = \begin{cases} > 0 & k < k^* \\ \leq 0 & k \geq k^* \end{cases}$$

Also, the law of motion in the Solow model describes changes in the capital level:

$$\dot{k} = s \cdot f(k) - \Delta(k) \cdot k$$

With a Cobb-Douglas production function, the law of motion results in the steady-state level of capital:

$$\dot{k} = s \cdot A \cdot k^{\alpha} - \delta \cdot k^{\beta} \cdot k = 0$$

We take the logarithms of the above equation and solve it for  $\ln(k^*)$  at the steady-state:

$$\ln(s \cdot A) + \alpha \cdot \ln(k^*) - \ln(\delta) - \beta \cdot \ln(k^*) = 0$$

$$\ln(k^*) = \frac{1}{\beta - \alpha} \cdot \ln(S \cdot A / \Delta)$$

The above equation presents a relationship between  $k^*$  and  $\beta$  in the steady state of  $\dot{k} = 0$ . For any level of  $k$ , if  $\beta$  is large enough, then high depreciation wears out capital and the capital stock would be on decline. On the contrary, if  $\beta$  is small, then lower depreciation leads to a positive change in capital stock. Combining  $\dot{k} = 0$  and  $\dot{\beta} = 0$  results in the following state diagram:

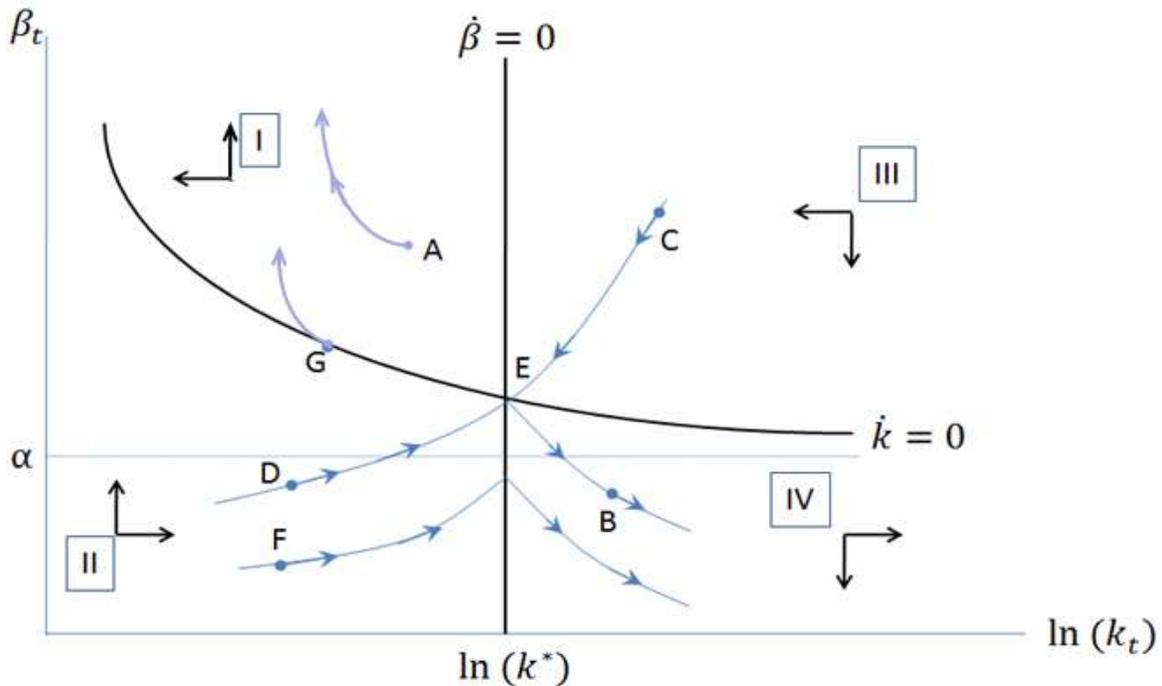


Figure 1. State diagram of  $\beta$  and  $\ln(k)$

Suppose that the economy is in zone I of the above state diagram. In this zone, the depreciation power,  $\beta$ , is above its equilibrium level and accordingly deprecation of capital would be excessive resulting declining level of capital. In addition, in this zone, the stock of capital is below its equilibrium level. Here, the “congestion effect” dominates; however as the capital stocks decline the “congestion effect” mitigates and  $\beta$  declines as well. The values of  $\beta$  and  $\ln(k)$  keeps declining until the economy reaches the point G on the curve  $\dot{k} = 0$ . Since capital stock remains unchanged along the curve  $\dot{k} = 0$ , escaping from this curve is cumbersome. Therefore all the points on the part of the curve  $\dot{k} = 0$  between zones I and II, create stable steady state equilibrium. We refer to this part of curve  $\dot{k} = 0$  as “depreciation trap.”

Notwithstanding, if the initial values of  $k$  and  $\beta$  are small and the value of  $\beta$  is relatively very low (points D and F in zone II) the dual growth of capital accumulation and depreciation power

would not end up into a “depreciation trap” as the economy enters zone IV before being engulfed in any trap.

In zone IV, the recycling effects dominate the congestion effects and therefore with capital accumulation, the depreciation power drops. Therefore as the economy grows, the value of  $\beta$  will approach its limit  $\alpha$ . In this case, the ultimate results resemble “endogenous growth models” in which an economy keeps growing unrestrictedly.

In zone III, both  $k$  and  $\beta$  are above their steady-state level, thus they drop over time. In this case for any certain level of capital if  $\beta$  is relatively lower, simultaneous drops of  $k$  and  $\beta$  lead the economy into everlasting growth of zone IV. On the contrary, for the same level of capital, if  $\beta$  is relatively higher, the economy will eventually enter the zone II and might end up in a crippling “depreciation trap.” Point C illustrates an intermediate state that concurrent drops in capital level and depreciation power drives the economy to the point G. It is notable that G is not a stable equilibrium point, rather it is a saddle point. From this point, any small efforts towards lower depreciation power and higher capital level impel the economy to the prosperous zone of IV. Thereafter, the economy will not return to the point G.

Gylfason and Zoega (2007) propose a static model to account for varied observed durability (depreciation) in the world. They conclude that increased population growth and rapid technological progress accelerate depreciation. This conclusion is based on their assumption of diminishing returns to durability, which is equivalent of dominance of “congestion effects” in our model. Here, the focus is on the dynamics of depreciation/ durability variations.

One finding of our model is that initial conditions have profound impact on the destiny of the nations towards either prosperous everlasting growth or vicious crippling stagnation. If the economy starts with relatively high depreciation power, then it cripples in “depreciation trap,” otherwise permanent growth is achievable.

## **Concluding remarks**

Accounting approaches to depreciation prevents growth economists to observe and investigate the existing varieties of depreciation rates among countries. Depreciation is a general concept that can be applied to any types of physical, human, social, or natural capital. In deed, depreciation rates among different economies could be remarkably different due to economic, geographical, and cultural factors.

In this model, we focus on different and sometimes opposing impacts of capital accumulation on depreciation function in the modern theory of growth. On one hand, capital accumulation can cause congestion, which in turn can escalate the depreciation power. We call this phenomenon as “congestion effects.” On the other hand, through capital accumulation, diversity of economic activities enhances and value chains are completed, reducing depreciation power through “recycling effects.”

One contribution of this paper is that by recognition of the importance of depreciation and combining its two determinant effects, remarkable explanatory power is created to describe both poverty traps and sustainable growth in one single model. Our dynamical model investigates the interaction between depreciation function and capital accumulation in a Solow-Swan growth model. The model emphasizes on the crucial role of initial conditions of depreciation factors. An economy that starts with low depreciation power and relatively higher level of capital can enter into an auspicious cycle of sustainable growth and declining depreciation power. In contrast, high depreciation power and relatively lower level of capital results in a special type of poverty trap: depreciation trap.

This paper is simply an initial step towards recognition of the role of quality of capital in economic growth. Although scarcity of capital might be held responsible for poor quality investment in developing country, this view appears simplistic when investigate this concept in resource-rich economies. Future studies need to explain other determinants of depreciation/ Quality of capital and their relevance to resource curse.

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